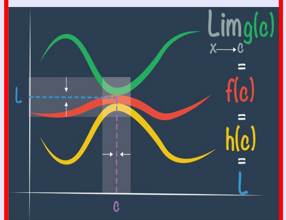
Math 261

Fall 2022

Lecture 21



Find $f'(x)$ and Solve $f'(x)=0$

$$f(x) = (4x - x^2)^{100}$$

$$f'(x) = 100(4x - x^2)^{99} \cdot (4 - 2x)$$

$f'(x) = 0$

$100 \neq 0$

$4x - x^2 = 0$

$x(4 - x) = 0$

$\boxed{x=0}$ $\boxed{x=4}$

$4 - 2x = 0$

$4 = 2x$

$\boxed{x=2}$

Find $f'(x)$ for $f(x) = a^3 + \cos^3 x^2$

$f'(x) = 0 + 3\cos^2 x^2 \cdot (-\sin(x^2)) \cdot 2x$

$$\boxed{f'(x) = -6x\cos^2 x^2 \cdot \sin x^2}$$

Find $f'(\theta)$ for $f(\theta) = \frac{3}{n}\sec\sqrt{n\theta}$

$f'(\theta) = \frac{3}{\cancel{n}} \cdot \sec\sqrt{n\theta}\,\tan\sqrt{n\theta} \cdot \frac{\cancel{n}}{2\sqrt{n\theta}}$

$\frac{d}{d\theta}\left[\sqrt{n\theta}\right] = \frac{d}{d\theta}\left[(n\theta)^{1/2}\right]$

$= \frac{1}{2} \cdot (n\theta)^{-1/2} \cdot n = \frac{n}{2\sqrt{n\theta}}$

$f'(\theta) = \frac{3}{2\sqrt{n\theta}}\sec\sqrt{n\theta}\tan\sqrt{n\theta}$

$\frac{d}{dx}\left[c\,f(x)\right] = c\,\frac{d}{dx}\left[f(x)\right]$

Find y' for $y = \frac{\cos\pi x}{\sin\pi x + \cos\pi x}$

$$y' = \frac{\frac{d}{dx}[\cos\pi x]\cdot(\sin\pi x + \cos\pi x) - \cos\pi x \cdot \frac{d}{dx}[\sin\pi x + \cos\pi x]}{(\sin\pi x + \cos\pi x)^2}$$

$$= \frac{-\sin\pi x \cdot \pi(\sin\pi x + \cos\pi x) - \cos\pi x[\cos\pi x \cdot \pi - \sin\pi x \cdot \pi]}{(\sin\pi x + \cos\pi x)^2}$$

$$y' = \frac{\pi\left[-\sin^2\pi x - \cancel{\sin\pi x\cos\pi x} - \cos^2\pi x + \cancel{\cos\pi x\sin\pi x}\right]}{(\sin\pi x + \cos\pi x)^2}$$

$$y' = \frac{\pi[-1]}{(\sin\pi x + \cos\pi x)^2} = \frac{-\pi}{(\sin\pi x + \cos\pi x)^2}$$

Find $\frac{dy}{dx}$ for $y = \underbrace{x \sin\frac{1}{x}}$ $\qquad \frac{d}{dx}\left[\frac{1}{x}\right] = \frac{-1}{x^2}$

$$\frac{dy}{dx} = 1 \cdot \sin\frac{1}{x} + \cancel{x} \cdot \cos\frac{1}{x} \cdot \frac{-1}{x^{\cancel{2}}}$$

$$\boxed{\frac{dy}{dx} = \sin\frac{1}{x} - \frac{1}{x}\cos\frac{1}{x}}$$

Find $f'(x)$ for $f(x) = \cancel{\sin}(\cancel{\cos}(\cancel{\tan}\, x))$

$$f'(x) = \cos(\cos(\tan x)) \cdot -\sin(\tan x) \cdot \sec^2 x \cdot 1$$

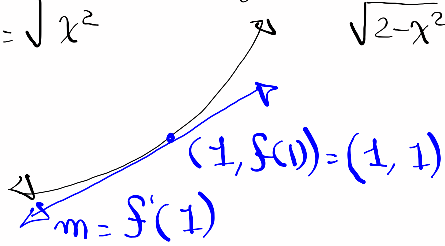
$$= -\cos(\cos(\tan x)) \cdot \sin(\tan x) \cdot \sec^2 x$$

Find eqn of tan. line to the curve given by $f(x) = \frac{|x|}{\sqrt{2-x^2}}$ at the point with $x = 1$.

$|x| = \sqrt{x^2}$

$$f(x) = \frac{\sqrt{x^2}}{\sqrt{2-x^2}} = \sqrt{\frac{x^2}{2-x^2}} = \left(\frac{x^2}{2-x^2}\right)^{1/2}$$

$$f'(x) = \frac{1}{2}\left(\frac{x^2}{2-x^2}\right)^{-1/2} \cdot \frac{4x}{(2-x^2)^2}$$



$$f'(1) = \frac{1}{2} \cdot 1 \cdot \frac{4}{1} = 2$$

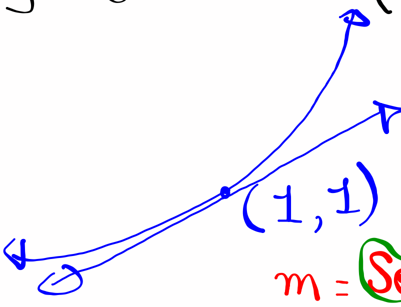
$$\frac{d}{dx}\left[\frac{x^2}{2-x^2}\right] = \frac{2x(2-x^2) - x^2 \cdot -2x}{(2-x^2)^2} = \frac{4x - 2x^3 + 2x^3}{(2-x^2)^2} = \frac{4x}{(2-x^2)^2}$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 2(x - 1) \rightarrow \boxed{y = 2x - 1}$$

Find eqn of tan. line to the curve given by $f(x) = \tan\left(\frac{\pi x^2}{4}\right)$ at the point with $x=1$.

$f(1) = \tan\left(\frac{\pi}{4}\right) = 1$



$m = f'(1)$

$f'(x) = \sec^2 \frac{\pi x^2}{4} \cdot \frac{\pi}{4} \cdot 2x$

$m = \sec^2\left(\frac{\pi}{4}\right) \cdot \frac{\pi}{4} \cdot 2$ $\qquad \sec \frac{\pi}{4} = \sqrt{2}$

$= (\sqrt{2})^2 \cdot \frac{\pi}{2} = 2 \cdot \frac{\pi}{2} = \boxed{\pi}$

$y - y_1 = m(x - x_1)$ $\qquad y - 1 = \pi(x - 1)$

$\boxed{y = \pi x + 1 - \pi}$

Show $\frac{d}{dx}\left[\,|x|\,\right] = \frac{x}{|x|}$

Hint: $|x| = \sqrt{x^2}$

$|x| = (x^2)^{\frac{1}{2}}$

$\frac{d}{dx}|x| = \frac{d}{dx}\left[(x^2)^{1/2}\right]$

$= \frac{1}{\cancel{2}} \cdot (x^2)^{-1/2} \cdot \cancel{2}x = \frac{x}{(x^2)^{1/2}} = \frac{x}{|x|}$

Show

$$\frac{d}{dx}\left(\sin^n x \cos nx\right) = n \sin^{n-1} x \cdot \cos(n+1)x$$

$$\frac{d}{dx}\left[\sin^n x \cdot \cos nx\right] = \frac{d}{dx}\left[\sin^n x\right] \cdot \cos nx + \sin^n x \cdot \frac{d}{dx}\left[\cos nx\right]$$

$$= n \sin^{n-1} x \cdot \cos x \cdot \cos nx + \sin^n x \cdot -\sin nx \cdot n$$

$$= n\left[\sin^{n-1} x \cos x \cdot \cos nx - \sin^n x \cdot \sin nx\right]$$

$$= n\left[\sin^{n-1} x \cdot \cos x \cdot \cos nx - \sin^{n-1} x \sin^1 x \cdot \sin nx\right]$$

$$= n \sin^{n-1} x\left[\cos x \cos nx - \sin x \sin nx\right]$$

Recall

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$= n \sin^{n-1} x \cdot \cos(x + nx)$$

$$= \boxed{n \sin^{n-1} x \cdot \cos(1+n)x}$$

Use the chart below to find $\frac{d}{dx}\left[f(g(x))\right]$ evaluated at 1.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x)$$

$$\text{at } x = 1 \Rightarrow f'(g(1)) \cdot g'(1) = f'(2) \cdot 6$$

$$= 5 \cdot 6 = \boxed{30}$$

If θ is measured in degrees,

Find $\frac{d}{d\theta}[\sin\theta]$

Hint: we always do radians in Calc.

$180^\circ = \pi$ Rad.

$1^\circ = \frac{\pi}{180}$ Rad.

$\theta^\circ = \frac{\pi}{180}\theta$ Rad.

$$\underbrace{\frac{d}{d\theta}[\sin\theta]}_{\text{Deg.}} = \underbrace{\frac{d}{d\theta}\left[\sin\frac{\pi}{180}\theta\right]}_{\text{Rad.}}$$

$$= \cos\frac{\pi}{180}\theta \cdot \frac{\pi}{180}$$

$$= \frac{\pi}{180}\cdot\underbrace{\cos\theta}_{\text{Deg.}}$$

Always do Radians in Calc.

Implicit Der.

$$\frac{d}{dx}\left[\sin\boxed{x^2}\right] = \cos x^2 \cdot 2x$$

$$\frac{d}{dx}\left[\boxed{\sin^{2}} x\right] = 2\sin x \cdot \cos x$$